only transverse shear and in the analysis the flexural rigidities of the face sheets about their own midplanes have been included.

Results for ω/ω_0 were again computed by omitting the effect of the rotary inertia from the governing differential equations. These results can be generated by redefining the four coefficients A_5, A_6, A_8, and A_9 in Eqs. (7) and then computing the roots of Eq. (14) again. For lower frequencies, the values obtained for ω/ω_0 by dropping the rotary inertia terms and the corresponding values shown in Fig. 2 are essentially the same. This conclusion is in agreement with the one for rectangular sandwich plates[5] and for homogeneous circular plates.[1] Rotary inertia terms would, however, play significant role for higher modes.

Figure 3 shows modal shapes for first three modes for which the frequency distribution is given in Fig. 2.

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Dependence of Plate-Bending Finite Element Deflections and Eigenvalues on Poisson's Ratio

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FOR uniform plates subjected only to a normal load and having boundaries consisting entirely of segments that are either a) clamped or b) simply supported and straight, the differential equations and boundary conditions depend on Poisson's ratio (ν) only indirectly (through the plate rigidity D). Hence, exact solutions for displacements and natural frequencies depend on Poisson's ratio only through D.[1] Cowper et al.[2] prove that finite element deflections and natural frequencies with conforming elements also depend on ν only through D. For nonconforming elements, however, the finite element approximations depend directly on ν even for the aforementioned class of problems of plate bending. No data are presently available in the literature on the extent of this dependence.

To study the effect of direct dependence on Poisson's ratio in the finite element approximations using nonconforming elements, numerical experiments were conducted on the static and free vibration analyses of a square plate with clamped and simply supported edges (see Fig. 1). Because of symmetry on the x- and y-axes, only one-quarter of the plate (shaded portion in Fig. 1a) is analyzed. The "P" and "Q" arrangements of mesh idealizations of the shaded one-quarter of plate are shown in Figs. 1b and 1c. Two problems of static analysis, viz., central concentrated load P and uniformly distributed load q on clamped square plate and one problem of free vibration analysis of simply supported square plate are worked out. The nonconforming triangular element of Narayanaswami[3,4] was used in the study. This noncompatible element has 18 degrees of freedom involving lateral displacement w and the two rotations, α and β, at the three corners and the three midside points. The interpolation function used is a quintic polynomial in x and y. Six values of Poisson's ratio were considered in the static analysis, viz., 0.0, 0.10, 0.20, 0.30, 0.40, and 0.495; and two values, viz., 0.0 and 0.3 in the free vibration analysis.

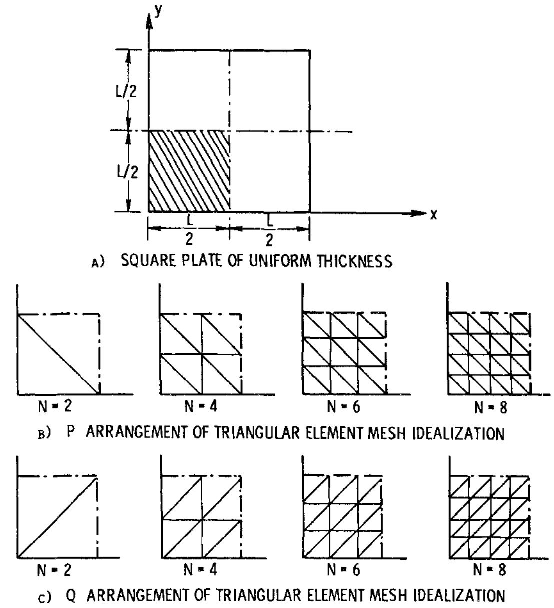

Fig. 1 Finite element idealization of square plate.

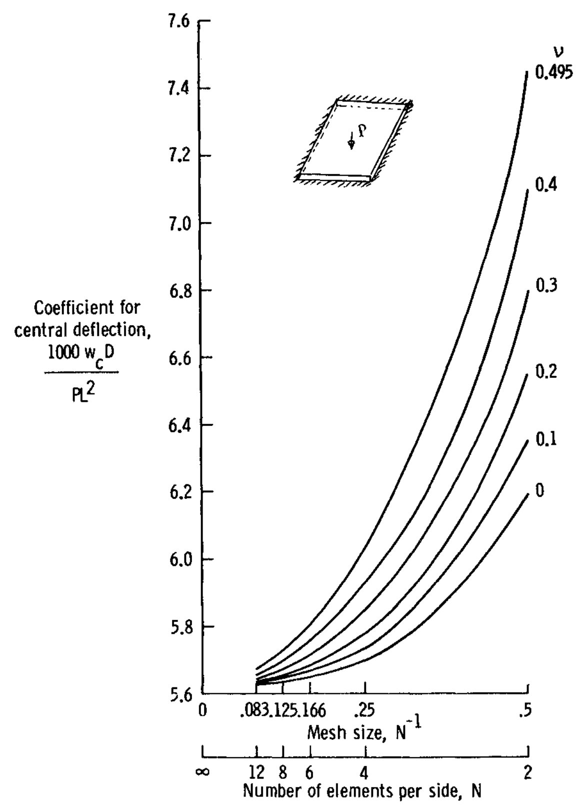

Fig. 2 Effect of Poisson's Ratio on nonconforming quintic Element.

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Table 1 Nondimensional eigenvalue numbers of simply supported square plate

Mode, (r,s)	Number of elements per side, N								Exact Solution
	2				4				
	ν = 0.0		ν = 0.3		ν = 0.0		ν = 0.3		
	Q Arrangement	P Arrangement	Q Arrangement	P Arrangement	Q Arrangement	P Arrangement	Q Arrangement	P Arrangement	
(1,1)	386.14	384.94	378.15	380.13	388.93	388.74	386.81	387.47	389.64
(1,2)	2477.59	2477.59	2476.96	2476.96	2415.26	2415.26	2386.91	2386.91	2435.23
(2,2)	6541.94	6541.94	6541.94	6541.94	6179.56	6179.56	6090.60	6090.60	6324.18
(1,3)	7149.37	10008.09	7361.34	9154.84	9482.08	9334.84	9060.72	9060.72	9740.91
	9922.22	10458.52	9967.93	9702.58	9704.72	9548.69	9499.83	9395.30	
(2,3)	18181.25	18181.25	17713.33	17713.33	16144.46	16144.46	15670.75	15670.75	16462.14
(3,3)	32072.08	35119.77	29521.72	32609.40	30843.18	30676.12	29507.27	29477.98	31560.55

The values of the nondimensional coefficients for central deflection for different Poisson's ratios for clamped plates for both central concentrated load ($w_c D/PL^2$) and uniformly distributed loads ($w_c D/qL^4$) are tabulated in Ref. 4. The results for the P mesh (see Fig. 1b) of a clamped square plate are plotted in Figs. 2 and 3. From Figs. 2 and 3, it is seen that the effect of an increase in the Poisson's ratio is to make the finite element approximation of the structure (using these elements) more flexible; and that, as the mesh is made finer and finer, the nondimensional coefficient tends to converge to the exact value for all values of v. Also, it can be noted that the percentage error in the coefficients is the least for $v = 0.0$.

The nondimensional eigenvalues for free vibration ($=\rho t\omega^2 L^4/D$) were evaluated for $v = 0.0$ and 0.3 for mesh size of $N = 2$ and $N = 4$ for the simply supported square plate. The results are shown in Table 1. Although the percentage error in the lowest eigenvalue is least when $v = 0.0$, the same is not true for other eigenvalues.

Based on this study, the following conclusions about the plate-bending finite element approximations using nonconforming elements in general, and the nonconforming element of Narayanaswami[3,4] in particular, are drawn:

1) The nondimensional coefficients for central deflection of a square plate, $w_c D/PL^2$ for central concentrated load and $w_c D/qL^4$ for uniformly distributed loads are dependent on Poisson's ratio, v.

2) The nondimensional eigenvalues, $\rho t\omega^2 L^4/D$, are also dependent on Poisson's ratio, v.

3) As the finite element mesh is made finer and finer, the coefficients tend to converge to the exact value for all values of v. Convergence depends on the problem and the elements used.

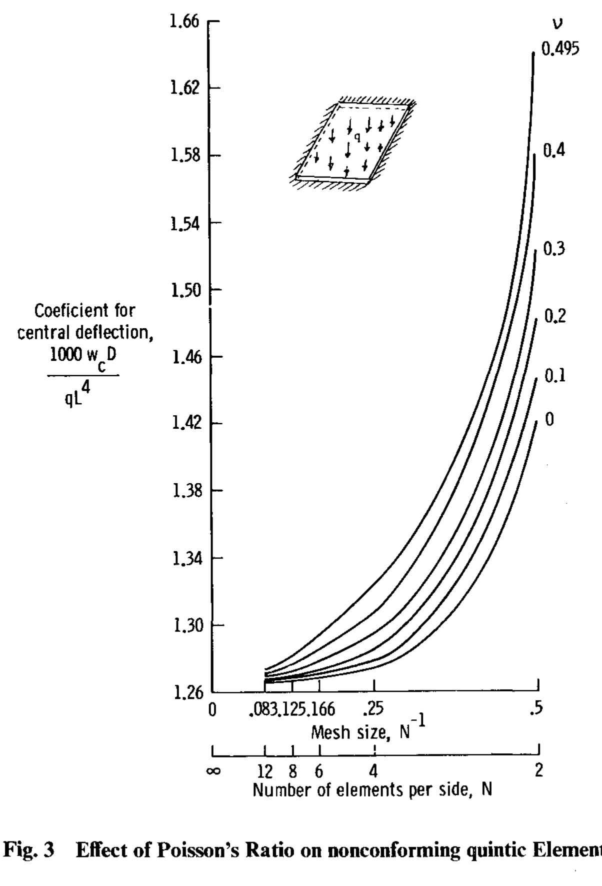

Fig. 3 Effect of Poisson's Ratio on nonconforming quintic Element.

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One-Parameter Solution of the Spanwise Rotating Blade Boundary-Layer Equation

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In Ref. 1 the author proposed a parametric method of solution of the spanwise boundary-layer equation for a long cylindrical blade. The one-parameter equation contained two

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